

Pensieve Header: A 2D B-Picture.

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SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\w-Computations"]
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C:\\drorbn\\AcademicPensieve\\Projects\\w-Computations
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```
HeadsMerge[x_, y_, z_][mix_] := Module[
  {cox, coy, jx, jy, jxy},
  cox = D[mix, h[x]] /. t[s_] => c[s];
  coy = D[mix, h[y]] /. t[s_] => c[s];
  jx = If[cox === 0, 1,  $\frac{-1 + e^{cox}}{cox}$ ];
  jy = If[coy === 0, 1,  $\frac{-1 + e^{coy}}{coy}$ ];
  jxy = If[cox + coy === 0, 1,  $\frac{-1 + e^{cox+coy}}{cox + coy}$ ];
  (mix /. {h[x] -> 0, h[y] -> 0}) +  $\frac{jx h[z] D[mix, h[x]]}{jxy}$  +  $e^{cox} \frac{jy h[z] D[mix, h[y]]}{jxy}$ 
]
```

```
HeadsMerge[3, 4, 5][t[1] h[3] + t[2] h[4]]
```

$$\frac{(-1 + e^{c[1]}) (c[1] + c[2]) h[5] t[1]}{(-1 + e^{c[1]+c[2]}) c[1]} + \frac{e^{c[1]} (-1 + e^{c[2]}) (c[1] + c[2]) h[5] t[2]}{(-1 + e^{c[1]+c[2]}) c[2]}$$

```
a = t[1] h[4] + t[2] h[5] + t[3] h[6]
```

```
h[4] t[1] + h[5] t[2] + h[6] t[3]
```

```
HeadsMerge[4, 5, 7][a]
```

$$\frac{(-1 + e^{c[1]}) (c[1] + c[2]) h[7] t[1]}{(-1 + e^{c[1]+c[2]}) c[1]} + \frac{e^{c[1]} (-1 + e^{c[2]}) (c[1] + c[2]) h[7] t[2]}{(-1 + e^{c[1]+c[2]}) c[2]} + h[6] t[3]$$

```
Expand2[expr_] := Series[expr /. {c[i_] => z c[i], t[i_] => z t[i]}, {z, 0, 2}]
```

```
HeadsMerge[4, 5, 7][a] // Expand2
```

$$(h[7] t[1] + h[7] t[2] + h[6] t[3]) z + \frac{1}{2} (-c[2] h[7] t[1] + c[1] h[7] t[2]) z^2 + O[z]^3$$

```
HeadsMerge[7, 6, 8][HeadsMerge[4, 5, 7][a]] // Simplify
```

$$\frac{((c[1] + c[2] + c[3]) h[8] (e^{c[1]} (-1 + e^{c[2]}) c[1] c[3] t[2] + c[2] ((-1 + e^{c[1]}) c[3] t[1] + e^{c[1]+c[2]} (-1 + e^{c[3]}) c[1] t[3]))))}{((-1 + e^{c[1]+c[2]+c[3]}) c[1] c[2] c[3])}$$

```
HeadsMerge[4, 7, 8][HeadsMerge[5, 6, 7][a]] // Simplify
```

$$\frac{((c[1] + c[2] + c[3]) h[8] (e^{c[1]} (-1 + e^{c[2]}) c[1] c[3] t[2] + c[2] ((-1 + e^{c[1]}) c[3] t[1] + e^{c[1]+c[2]} (-1 + e^{c[3]}) c[1] t[3]))))}{((-1 + e^{c[1]+c[2]+c[3]}) c[1] c[2] c[3])}$$

```
(HeadsMerge[7, 6, 8][HeadsMerge[4, 5, 7][a]] -
```

```
HeadsMerge[4, 7, 8][HeadsMerge[5, 6, 7][a]]) // Simplify
```

0

**a = Sum[Random[] t[i] h[j], {i, 4}, {j, 4}]**

0.0752672 h[1] t[1] + 0.226776 h[2] t[1] + 0.912243 h[3] t[1] + 0.154449 h[4] t[1] +  
 0.945915 h[1] t[2] + 0.267741 h[2] t[2] + 0.646716 h[3] t[2] + 0.70167 h[4] t[2] +  
 0.757047 h[1] t[3] + 0.815404 h[2] t[3] + 0.413309 h[3] t[3] + 0.0123457 h[4] t[3] +  
 0.518825 h[1] t[4] + 0.645992 h[2] t[4] + 0.746541 h[3] t[4] + 0.190113 h[4] t[4]

**Simplify[HeadsMerge[1, 2, 6][a]]**

0. + 0.912243 h[3] t[1] + 0.154449 h[4] t[1] +  
 0.646716 h[3] t[2] + 0.70167 h[4] t[2] + 0.413309 h[3] t[3] +  
 0.0123457 h[4] t[3] +  $\left( (-1. + e^{0.0752672 c[1] + 0.945915 c[2] + 0.757047 c[3] + 0.518825 c[4]}) \right)$   
 $\left( (0.302043 c[1] + 1.21366 c[2] + 1.57245 c[3] + 1.16482 c[4]) h[6] \right)$   
 $\left( (0.0752672 t[1] + 0.945915 t[2] + 0.757047 t[3] + 0.518825 t[4]) \right) /$   
 $\left( (-1. + e^{0.302043 c[1] + 1.21366 c[2] + 1.57245 c[3] + 1.16482 c[4]}) \right)$   
 $\left( (0.0752672 c[1] + 0.945915 c[2] + 0.757047 c[3] + 0.518825 c[4]) \right) +$   
 $\left( e^{0.0752672 c[1] + 0.945915 c[2] + 0.757047 c[3] + 0.518825 c[4]} \right)$   
 $\left( (-1. + e^{0.226776 c[1] + 0.267741 c[2] + 0.815404 c[3] + 0.645992 c[4]}) \right)$   
 $\left( (0.302043 c[1] + 1.21366 c[2] + 1.57245 c[3] + 1.16482 c[4]) h[6] \right)$   
 $\left( (0.226776 t[1] + 0.267741 t[2] + 0.815404 t[3] + 0.645992 t[4]) \right) /$   
 $\left( (-1. + e^{0.302043 c[1] + 1.21366 c[2] + 1.57245 c[3] + 1.16482 c[4]}) \right)$   
 $\left( (0.226776 c[1] + 0.267741 c[2] + 0.815404 c[3] + 0.645992 c[4]) \right) +$   
 0.746541 h[3] t[4] + 0.190113 h[4] t[4]

**t1 = Simplify[HeadsMerge[6, 3, 7][Simplify[HeadsMerge[1, 2, 6][a]]]**

0. + 0.154449 h[4] t[1] + 0.70167 h[4] t[2] + 0.0123457 h[4] t[3] +  
 $\left( (-1. + e^{0.302043 c[1] + 1.21366 c[2] + 1.57245 c[3] + 1.16482 c[4]}) \right) \left( -1. + 1. \right)$   
 $\frac{\left( (-0.302043 + 0.302043 e^{0.302043 c[1] + 1.21366 c[2] + 1.57245 c[3] + 1.16482 c[4]}) c[1] + (-1.21366 + 1.21366 e^{0.302043 c[1] + 1.21366 c[2] + 1.57245 c[3] + 1.16482 c[4]}) c[2] - 1.57245 c[3] + 1.16482 c[4] \right)}{-1. + e^{0.302043 c[1] + 1.21366 c[2] + 1.57245 c[3] + 1.16482 c[4]}}$   
 $\left( (0. + 0.912243 c[1] + 0.646716 c[2] + 0.413309 c[3] + \right.$   
 $0.746541 c[4] + \left. \left( (-1. + e^{0.0752672 c[1] + 0.945915 c[2] + 0.757047 c[3] + 0.518825 c[4]}) \right) \right)$   
 $\left( (0.302043 c[1] + 1.21366 c[2] + 1.57245 c[3] + 1.16482 c[4]) \right) /$   
 $\left( (-1. + e^{0.302043 c[1] + 1.21366 c[2] + 1.57245 c[3] + 1.16482 c[4]}) \right) +$   
 $\left( e^{0.0752672 c[1] + 0.945915 c[2] + 0.757047 c[3] + 0.518825 c[4]} \right)$   
 $\left( (-1. + e^{0.226776 c[1] + 0.267741 c[2] + 0.815404 c[3] + 0.645992 c[4]}) \right)$   
 $\left( (0.302043 c[1] + 1.21366 c[2] + 1.57245 c[3] + 1.16482 c[4]) \right) /$   
 $\left( (-1. + e^{0.302043 c[1] + 1.21366 c[2] + 1.57245 c[3] + 1.16482 c[4]}) \right) h[7]$   
 $\left( (0. + \left( (-1. + e^{0.0752672 c[1] + 0.945915 c[2] + 0.757047 c[3] + 0.518825 c[4]}) \right) \right)$   
 $\left( (0.302043 c[1] + 1.21366 c[2] + 1.57245 c[3] + 1.16482 c[4]) \right)$   
 $\left( (0.0752672 t[1] + 0.945915 t[2] + 0.757047 t[3] + 0.518825 t[4]) \right) /$   
 $\left( (-1. + e^{0.302043 c[1] + 1.21366 c[2] + 1.57245 c[3] + 1.16482 c[4]}) \right)$   
 $\left( (0.0752672 c[1] + 0.945915 c[2] + 0.757047 c[3] + 0.518825 c[4]) \right) +$   
 $\left( e^{0.0752672 c[1] + 0.945915 c[2] + 0.757047 c[3] + 0.518825 c[4]} \right)$   
 $\left( (-1. + e^{0.226776 c[1] + 0.267741 c[2] + 0.815404 c[3] + 0.645992 c[4]}) \right)$   
 $\left( (0.302043 c[1] + 1.21366 c[2] + 1.57245 c[3] + 1.16482 c[4]) \right)$   
 $\left( (0.226776 t[1] + 0.267741 t[2] + 0.815404 t[3] + 0.645992 t[4]) \right) /$   
 $\left( (-1. + e^{0.302043 c[1] + 1.21366 c[2] + 1.57245 c[3] + 1.16482 c[4]}) \right)$

$$\begin{aligned}
& \left. \left( 0.226776 c[1] + 0.267741 c[2] + 0.815404 c[3] + 0.645992 c[4] \right) \right) / \left( \left( -1. + 1. \right. \right. \\
& \left. \left. \frac{(-1.21429+1.21429 e^{0.302043 c[1]}+1.21366 c[2]-1.57245 c[3]+1.16482 c[4]) c[1]+(-1.86037+1.86037 e^{0.302043 c[1]}+1.21366 c[2]-1.57245 c[3]+1.16482 c[4]) c[2]-1.98576 c[3]+1.98576 e^{0.302043 c[1]}+1.21366 c[2]-1.57245 c[3]+1.16482 c[4]) c[3]+(-1.86037+1.86037 e^{0.302043 c[1]}+1.21366 c[2]-1.57245 c[3]+1.16482 c[4]) c[4]}{-1.+e^{0.302043 c[1]}+1.21366 c[2]-1.57245 c[3]+1.16482 c[4]} \right) \right) \\
& \left. \left( 0. + \left( -0.302043 + 0.302043 e^{0.302043 c[1]}+1.21366 c[2]+1.57245 c[3]+1.16482 c[4] \right) \right) \right) \\
& c[1] + \left( -1.21366 + 1.21366 e^{0.302043 c[1]}+1.21366 c[2]+1.57245 c[3]+1.16482 c[4] \right) c[2] - \\
& 1.57245 c[3] + 1.57245 e^{0.302043 c[1]}+1.21366 c[2]+1.57245 c[3]+1.16482 c[4] c[3] - \\
& 1.16482 c[4] + 1.16482 e^{0.302043 c[1]}+1.21366 c[2]+1.57245 c[3]+1.16482 c[4] c[4] \left. \right) + \left( 1. \right. \\
& \left. \frac{0.+(-0.302043+0.302043 e^{0.302043 c[1]}+1.21366 c[2]-1.57245 c[3]+1.16482 c[4]) c[1]+(-1.21366+1.21366 e^{0.302043 c[1]}+1.21366 c[2]-1.57245 c[3]+1.16482 c[4]) c[2]-1.57245 c[3]+1.57245 e^{0.302043 c[1]}+1.21366 c[2]-1.57245 c[3]+1.16482 c[4]) c[3]+(-1.86037+1.86037 e^{0.302043 c[1]}+1.21366 c[2]-1.57245 c[3]+1.16482 c[4]) c[4]}{-1.+e^{0.302043 c[1]}+1.21366 c[2]-1.57245 c[3]+1.16482 c[4]} \right) \\
& \left( -1. + e^{0.912243 c[1]}+0.646716 c[2]+0.413309 c[3]+0.746541 c[4] \right) \\
& \left( 0. + 0.912243 c[1] + 0.646716 c[2] + 0.413309 c[3] + 0.746541 c[4] + \right. \\
& \left. \left( (-1. + e^{0.0752672 c[1]}+0.945915 c[2]+0.757047 c[3]+0.518825 c[4]) \left( 0.302043 c[1] + 1.21366 c[2] + \right. \right. \right. \\
& \left. \left. \left. 1.57245 c[3] + 1.16482 c[4] \right) \right) / \left( -1. + e^{0.302043 c[1]}+1.21366 c[2]+1.57245 c[3]+1.16482 c[4] \right) + \right. \\
& \left. \left( e^{0.0752672 c[1]}+0.945915 c[2]+0.757047 c[3]+0.518825 c[4] \right) \right. \\
& \left. \left( -1. + e^{0.226776 c[1]}+0.267741 c[2]+0.815404 c[3]+0.645992 c[4] \right) \right. \\
& \left. \left( 0.302043 c[1] + 1.21366 c[2] + 1.57245 c[3] + 1.16482 c[4] \right) \right) / \\
& \left. \left( -1. + e^{0.302043 c[1]}+1.21366 c[2]+1.57245 c[3]+1.16482 c[4] \right) \right) h[7] \\
& \left. \left( 0.912243 t[1] + 0.646716 t[2] + 0.413309 t[3] + 0.746541 t[4] \right) \right) / \left( \left( -1. + 1. \right. \right. \\
& \left. \left. \frac{(-1.21429+1.21429 e^{0.302043 c[1]}+1.21366 c[2]-1.57245 c[3]+1.16482 c[4]) c[1]+(-1.86037+1.86037 e^{0.302043 c[1]}+1.21366 c[2]-1.57245 c[3]+1.16482 c[4]) c[2]-1.98576 c[3]+1.98576 e^{0.302043 c[1]}+1.21366 c[2]-1.57245 c[3]+1.16482 c[4]) c[3]+(-1.86037+1.86037 e^{0.302043 c[1]}+1.21366 c[2]-1.57245 c[3]+1.16482 c[4]) c[4]}{-1.+e^{0.302043 c[1]}+1.21366 c[2]-1.57245 c[3]+1.16482 c[4]} \right) \right) \\
& \left. \left( 0.912243 c[1] + 0.646716 c[2] + 0.413309 c[3] + 0.746541 c[4] \right) \right) + \\
& 0.190113 h[4] t[4] \\
\mathbf{t2 = Simplify[HeadsMerge[1, 6, 7][Simplify[HeadsMerge[2, 3, 6][a]]]} \\
0. + 0.154449 h[4] t[1] + 0.70167 h[4] t[2] + \\
0.0123457 h[4] t[3] + \left( e^{0.0752672 c[1]}+0.945915 c[2]+0.757047 c[3]+0.518825 c[4] \right) \\
\left( -1. + e^{1.13902 c[1]}+0.914457 c[2]+1.22871 c[3]+1.39253 c[4] \right) \left( -1. + 1. \right. \\
\left. \frac{(-1.13902+1.13902 e^{1.13902 c[1]}+0.914457 c[2]-1.22871 c[3]+1.39253 c[4]) c[1]+(-0.914457+0.914457 e^{1.13902 c[1]}+0.914457 c[2]-1.22871 c[3]+1.39253 c[4]) c[2]-1.22871 c[3]+1.22871 e^{1.13902 c[1]}+0.914457 c[2]-1.22871 c[3]+1.39253 c[4]) c[3]+(-0.914457+0.914457 e^{1.13902 c[1]}+0.914457 c[2]-1.22871 c[3]+1.39253 c[4]) c[4]}{-1.+e^{1.13902 c[1]}+0.914457 c[2]-1.22871 c[3]+1.39253 c[4]} \right) \\
\left. \left( 0. + 0.0752672 c[1] + 0.945915 c[2] + 0.757047 c[3] + \right. \right. \\
0.518825 c[4] + \left. \left( (-1. + e^{0.226776 c[1]}+0.267741 c[2]+0.815404 c[3]+0.645992 c[4]) \right) \right. \\
\left. \left( 1.13902 c[1] + 0.914457 c[2] + 1.22871 c[3] + 1.39253 c[4] \right) \right) / \\
\left( -1. + e^{1.13902 c[1]}+0.914457 c[2]+1.22871 c[3]+1.39253 c[4] \right) + \\
\left( e^{0.226776 c[1]}+0.267741 c[2]+0.815404 c[3]+0.645992 c[4] \right) \\
\left( -1. + e^{0.912243 c[1]}+0.646716 c[2]+0.413309 c[3]+0.746541 c[4] \right) \\
\left( 1.13902 c[1] + 0.914457 c[2] + 1.22871 c[3] + 1.39253 c[4] \right) \left. \right) /
\end{aligned}$$

$$\begin{aligned}
 & \left( -1. + e^{1.13902 c[1]+0.914457 c[2]+1.22871 c[3]+1.39253 c[4]} \right) h[7] \\
 & \left( 0. + \left( \left( -1. + e^{0.226776 c[1]+0.267741 c[2]+0.815404 c[3]+0.645992 c[4]} \right) \right. \right. \\
 & \quad \left. \left( 1.13902 c[1] + 0.914457 c[2] + 1.22871 c[3] + 1.39253 c[4] \right) \right. \\
 & \quad \left. \left( 0.226776 t[1] + 0.267741 t[2] + 0.815404 t[3] + 0.645992 t[4] \right) \right) / \\
 & \left( \left( -1. + e^{1.13902 c[1]+0.914457 c[2]+1.22871 c[3]+1.39253 c[4]} \right) \right. \\
 & \quad \left. \left( 0.226776 c[1] + 0.267741 c[2] + 0.815404 c[3] + 0.645992 c[4] \right) \right) + \\
 & \left( e^{0.226776 c[1]+0.267741 c[2]+0.815404 c[3]+0.645992 c[4]} \right. \\
 & \quad \left( -1. + e^{0.912243 c[1]+0.646716 c[2]+0.413309 c[3]+0.746541 c[4]} \right) \\
 & \quad \left( 1.13902 c[1] + 0.914457 c[2] + 1.22871 c[3] + 1.39253 c[4] \right) \\
 & \quad \left. \left( 0.912243 t[1] + 0.646716 t[2] + 0.413309 t[3] + 0.746541 t[4] \right) \right) / \\
 & \left( \left( -1. + e^{1.13902 c[1]+0.914457 c[2]+1.22871 c[3]+1.39253 c[4]} \right) \right. \\
 & \quad \left. \left( 0.912243 c[1] + 0.646716 c[2] + 0.413309 c[3] + 0.746541 c[4] \right) \right) \Bigg/ \left( \left( -1. + 1. \right. \right. \\
 & \quad \left. \left. \frac{(-1.21429+1.21429 e^{1.13902 c[1]+0.914457 c[2]+1.22871 c[3]+1.39253 c[4]}) c[1]+(-1.86037+1.86037 e^{1.13902 c[1]+0.914457 c[2]+1.22871 c[3]+1.39253 c[4]}) c[2]-1.98576 c[3]+1.98576 e^1}{-1.+e^{1.13902 c[1]+0.914457 c[2]+1.22871 c[3]+1.39253 c[4]}} \right) \right) \\
 & \left. \left( 0. + \left( -1.13902 + 1.13902 e^{1.13902 c[1]+0.914457 c[2]+1.22871 c[3]+1.39253 c[4]} \right) c[1] + \right. \\
 & \quad \left( -0.914457 + 0.914457 e^{1.13902 c[1]+0.914457 c[2]+1.22871 c[3]+1.39253 c[4]} \right) c[2] - \\
 & \quad 1.22871 c[3] + 1.22871 e^{1.13902 c[1]+0.914457 c[2]+1.22871 c[3]+1.39253 c[4]} c[3] - \\
 & \quad \left. 1.39253 c[4] + 1.39253 e^{1.13902 c[1]+0.914457 c[2]+1.22871 c[3]+1.39253 c[4]} c[4] \right) \Bigg) + \\
 & \left( -1 + e^{0.0752672 c[1]+0.945915 c[2]+0.757047 c[3]+0.518825 c[4]} \right) \\
 & \left( 0. + 0.0752672 c[1] + 0.945915 c[2] + 0.757047 c[3] + 0.518825 c[4] + \right. \\
 & \quad \left( \left( -1. + e^{0.226776 c[1]+0.267741 c[2]+0.815404 c[3]+0.645992 c[4]} \right) \left( 1.13902 c[1] + 0.914457 c[2] + \right. \right. \\
 & \quad \quad \left. \left. 1.22871 c[3] + 1.39253 c[4] \right) \right) / \left( -1. + e^{1.13902 c[1]+0.914457 c[2]+1.22871 c[3]+1.39253 c[4]} \right) + \\
 & \quad \left( e^{0.226776 c[1]+0.267741 c[2]+0.815404 c[3]+0.645992 c[4]} \right. \\
 & \quad \left( -1. + e^{0.912243 c[1]+0.646716 c[2]+0.413309 c[3]+0.746541 c[4]} \right) \\
 & \quad \left. \left( 1.13902 c[1] + 0.914457 c[2] + 1.22871 c[3] + 1.39253 c[4] \right) \right) / \\
 & \quad \left( -1. + e^{1.13902 c[1]+0.914457 c[2]+1.22871 c[3]+1.39253 c[4]} \right) \Bigg) h[7] \\
 & \left( 0.0752672 t[1] + 0.945915 t[2] + 0.757047 t[3] + 0.518825 t[4] \right) \Bigg/ \left( \left( -1. + 1. \right. \right. \\
 & \quad \left. \left. \frac{(-1.21429+1.21429 e^{1.13902 c[1]+0.914457 c[2]+1.22871 c[3]+1.39253 c[4]}) c[1]+(-1.86037+1.86037 e^{1.13902 c[1]+0.914457 c[2]+1.22871 c[3]+1.39253 c[4]}) c[2]-1.98576 c[3]+1.98576 e^1}{-1.+e^{1.13902 c[1]+0.914457 c[2]+1.22871 c[3]+1.39253 c[4]}} \right) \right) \\
 & \left. \left( 0.0752672 c[1] + 0.945915 c[2] + 0.757047 c[3] + 0.518825 c[4] \right) \right) \Bigg) + \\
 & 0.190113 h[4] t[4]
 \end{aligned}$$

**Simplify[**

**t1 - t2 /. {c[1] → Random[], c[2] → Random[], c[3] → Random[], c[4] → Random[]}**

0. +

**h[7] (-2.22045 × 10<sup>-15</sup> t[1] - 1.55431 × 10<sup>-15</sup> t[2] - 1.33227 × 10<sup>-15</sup> t[3] - 2.44249 × 10<sup>-15</sup> t[4])**